**Topics: Normal distribution, Functions of Random Variables**

1. The time required for servicing transmissions is normally distributed with *μ* = 45 minutes and *σ* = 8 minutes. The service manager plans to have work begin on the transmission of a customer’s car 10 minutes after the car is dropped off and the customer is told that the car will be ready within 1 hour from drop-off. What is the probability that the service manager cannot meet his commitment?
2. 0.3875
3. 0.2676
4. 0.5
5. 0.6987

**Ans: B) 0.2676**

We have the normal distribution with = 45 and = 8.0 . Let X be the amount of time it

Takes to complete the repair on a customer’s car. To finish in one hour you must have

X <= 50 so the question is to find Pr(X>50).

Pr(X>50) = 1 – Pr(X <= 50).

Z = (X-)/ = (X-45)/8.0

Thus the question can be answered by using the normal table to find

Pr(X <= 50) = Pr(Z <= (50-45)/8.0) = Pr(Z <= 0.625) = 73.4%.

Probability that the service manager will not meet his demand will be

= 100 – 73.4

= 26.6% or 0.2676.

1. The current age (in years) of 400 clerical employees at an insurance claims processing center is normally distributed with mean *μ* = 38 and Standard deviation *σ* =6. For each statement below, please specify True/False. If false, briefly explain why.
2. More employees at the processing center are older than 44 than between 38 and 44.
3. A training program for employees under the age of 30 at the center would be expected to attract about 36 employees.

**Ans:**

We have a nomal distribution with = 38 and = 6.

Let X be the number of employees. so according to question.

1. Probability of employees greater than age of 44 = Pr(X>44)

Pr (X>44) = 1 – Pr(X <= 44).

Z = (X - )/ = (X – 38)/6

Thus the question can be answered by using the normal table to find

Pr(X <= 44) = Pr(Z <= (44-38)/6) = Pr(Z <= 1) = 84.1345%

Probability that the employee will be greater than age of 44

= 100 – 84.1345

= 15.86%

So the probability of number of employees between 38 – 44 years of age

= Pr(X < 44) – 0.5

= 84.1345 – 0.5

= 34.1345%

Therefore the statement that “ More employees at the processing center are older than 44 than between 38 and 44” is TRUE.

1. Probability of employees less than age of 30 = Pr(X < 30)

Z = (X-)/ = (30-38)/6

Thus the question can be answered by using the normal table to find

Pr(X <= 30) = Pr(Z <= (30-38)/6)

= Pr(Z <= -1.333)

= 9.12%

So the number of employees with probability 0.912 of them being under age 30

= 0.0912\*400

= 36.48 (or 36 employees)

Therefore the statement B of the question is also True.

1. If *X1* ~ *N*(μ, σ2) and *X*2 ~ *N*(μ, σ2) are *iid* normal random variables, then what is the difference between 2 *X*1 and *X*1 + *X*2? Discuss both their distributions and parameters.

**Ans:**

As we know that if X ∼ N(μ1, σ1^2 ), and Y ∼ N(μ2, σ2^2 ) are two independent random variables then X + Y ∼ N(μ1 + μ2, σ1^2 + σ2^2 ) , and X − Y ∼ N(μ1 − μ2, σ1^2 + σ2^2 ) .

Similarly if Z = aX + bY , where X and Y are as defined above, i.e Z is linear combination of X and Y , then Z ∼ N(aμ1 + bμ2, a^2σ1^2 + b^2σ2^2 ).

Therefore in the question

2X1~ N(2 u,4 σ^2) and

X1+X2 ~ N(μ + μ, σ^2 + σ^2 ) ~ N(2 u, 2σ^2 )

2X1-(X1+X2) = N( 4μ,6 σ^2).

1. Let X ~ N(100, 202). Find two values, *a* and *b*, symmetric about the mean, such that the probability of the random variable taking a value between them is 0.99.
2. 90.5, 105.9
3. 80.2, 119.8
4. 22, 78
5. 48.5, 151.5
6. 90.1, 109.9

**Ans:** D) 48.5, 151.5

Since we need to find out the values of a and b , which are symmetric about the mean , such that the probability of random variable taking a value between them is 0.99 . we have to work out in reverse order.

The probability of getting value between a and b should be 0.99

So the probability of going wrong, or the probability outside the a and b area is 0.01 (i.e 1-0.99)

The probability towards left from a = -0.005 (i.e 0.01/2).

The probability towards right from b = +0.005 (i.e. 0.01/2).

So since we have the probabilities of a and b , we need to calculate X , the random variable at a and b which has got these probabilities .

By finding the standard Normal Variable Z (Z value) , we can calculate the X values.

Z = (X – u ) / sigma

For probability 0.005 the Z value is -2.57 (From Z table)

Z\*sigma + u = X

Z(-0.005)\*20 +100 = -(-2.57)\*20 + 100 = 151.5

Z(-0.005)\*20 +100 = (-2.57)\*20 + 100 = 48.5

So, option D is correct .

1. Consider a company that has two different divisions. The annual profits from the two divisions are independent and have distributions Profit1 ~ N(5, 32) and Profit2 ~ N(7, 42) respectively. Both the profits are in $ Million. Answer the following questions about the total profit of the company in Rupees. Assume that $1 = Rs. 45
2. Specify a Rupee range (centered on the mean) such that it contains 95% probability for the annual profit of the company.
3. Specify the 5th percentile of profit (in Rupees) for the company
4. Which of the two divisions has a larger probability of making a loss in a given year?



